Math Virtual Learning

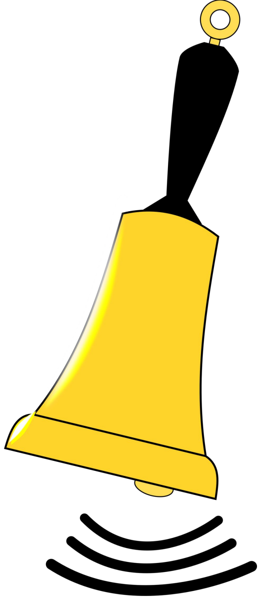
Probability and Statistics

April 30, 2020

Probability and Statistics
Lesson: April 30, 2020

Objective/Learning Target:
Students will use the Central Limit Theorem to predict the probability of sample means

Let's Get Started!



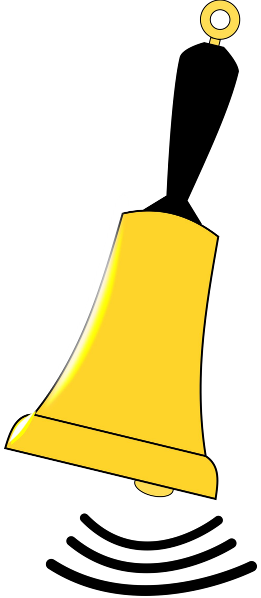
Use the Z-Score Conversion Chart to answer the following questions.

A sample of bees had their wingspan measured (in mm). It was found that the sample has a mean of 22 and a standard deviation of 4.

What percent of the bees have a wingspan between 16 and 23?

What percent of bees have a wingspan less than 18?

Let’s Get Started! ANSWERS



Use the Z-Score Conversion Chart to answer the following questions.

What percent of the bees have a wingspan between 16 and 23?

$$z = \frac{16-22}{4} = \frac{-6}{4} = -1.5$$

-1.5 = 6.68%

59.87 - 6.68 = 53.19%

$$z = \frac{23-22}{4} = \frac{1}{4} = 0.25$$

0.25 = 59.87%

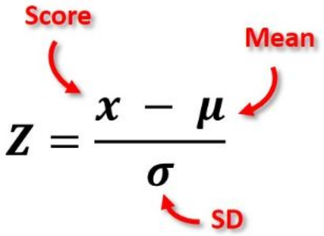
What percent of bees have a wingspan less than 18?

18 is one standard deviation away, so it has a z-score of 1. The percent is 84%

Z-score Recap

Things to Remember:

- We used z-scores to find percentages of data that fall into a certain range
- We used the formula



$$Z = \frac{x - \mu}{\sigma}$$

- We used this Conversion Chart to translate a z-score into a percentage
- This was for **specific data points** of a **specific sample** that followed a normal curve

Central Limit Theorem Recap

Things to Remember…

- Not all data samples are normal, but that doesn't make them completely invalid or useless
- The Central Limit says that even if we can't calculate data on a sample, we can calculate data on the **MEANS** of random samples taken over & over again.
  - Example - If we sampled the weight of 25 boys at a middle school, the data may not be normal. BUT if we sample another 25 and then another 25 and then another 25 over and over again, the MEANS of all those samples would be normal
- The Population Mean will always be the same as the Sample Mean
- The Population standard deviation and the Sample standard deviation are NOT the same

Using Z-Scores with the Central Limit Theorem

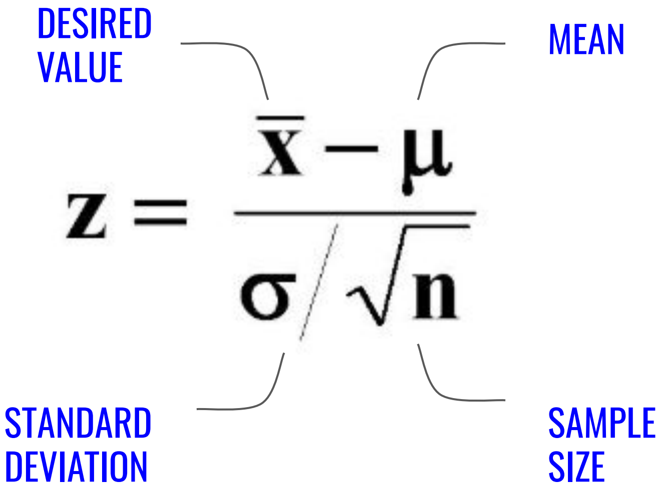
- We may want to know the probability that a sample mean will fall into a certain range.
    - Example - What are the chances (probability) that a random sample of 25 middle school boys will have an average weight between 125-136 pounds?
- Before we were looking for the actual percent of our sample that fell in that range
- Now we are looking to determine the likelihood that the average will be in a certain range
- Because we are still looking for percentages, but of something slightly different (the mean vs. actual number of people) - our Z-Score formula will be slightly different

Z-Score Formula when using the CLT

This formula is almost identical to the one we used before. But because we are focusing on the MEANS, there is an added piece in the denominator.

Instead of dividing only by the Standard Deviation, we have to first divide the Standard Deviation by the Square Root of the Sample Size.

Then work the formula like normal



DESIRED VALUE

MEAN

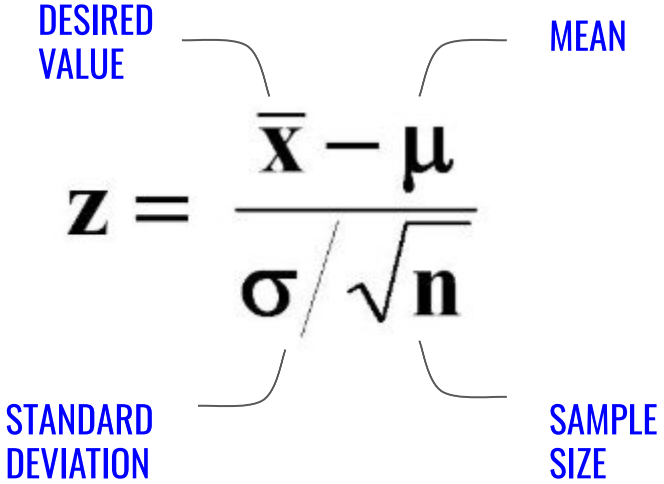
$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

STANDARD DEVIATION

SAMPLE SIZE

Example

A local research company investigated the number of times a person receives a spam phone call in a given month. The population had a mean of 12 and a standard deviation of 5. What is the probability (chance) that a random sample of 144 people will receive an average of 11-13 spam phone calls per month?



$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{11 - 12}{\frac{5}{\sqrt{144}}} = \frac{-1}{0.417} = -2.4$$

$$z = \frac{13 - 12}{\frac{5}{\sqrt{144}}} = \frac{1}{0.417} = 2.4$$

Example

A local research company investigated the number of times a person receives a spam phone call in a given month. The population had a mean of 12 and a standard deviation of 5. What is the probability (chance) that a random sample of 144 people will receive an average of 11-13 spam phone calls per month?

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Using the Conversion Chart

- -2.4 = 0.0082 = 0.82%
- 2.4 = 0.9918 = 99.18%
- Chances that it is between 11-13 would be 99.18-0.82 = 98.36%

So there is a 98.36% probability that if 144 people were surveyed they would have an **AVERAGE** of 11-13 spam phone calls per month.

For the next practice problems, you will need:

Z-Score to Percent Chart **Z-Score to Percent Chart**

Sample Mean = Pop Mean: $\mu_{\bar{x}} = \mu$

Sample S.D. uses this formula: $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Z-Score for Sample Distribution
Uses this formula: $\dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

All practice is taken from this LINK but has been modified for our needs:

1. A bottling company uses a filling machine to fill plastic bottles with a popular cola. The bottles are supposed to contain 300 milliliters (ml). In fact, the contents vary according to a normal distribution with mean m = 303 ml and standard deviation s = 3 ml.
    a. What is the probability that an individual bottle contains less than 300 ml?
    b. Now take a random sample of 10 bottles. What are the mean and standard deviation of the sample mean contents x-bar of these 10 bottles?
    c. What is the probability that the sample mean contents of the 10 bottles is less than 300 ml?

Click Here for Answers

#2

The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with mean 264 days and standard deviation 16 days. Consider 15 pregnant women from a rural area. Assume they are equivalent to a random sample from all women.

a. What are the mean and standard deviation of the sample mean length of pregnancy x-bar of these 15 pregnancies?

b. What's the probability the sample mean length of pregnancy lasts less than 250 days?

Click Here for Answers

#3

The weights of the eggs produced by a certain breed of hen are normally distributed with mean 65 grams and standard deviation of 5 grams.

a. What is the probability that one egg selected at random from a hen house will weigh <u>more than</u> 68 grams?
b. Consider a carton of 12 eggs to be a simple random sample (SRS) of hen's eggs. If you were to take a large number of repeated samples of size n = 12, what would the mean and standard deviation be of these sample means?
c. What is the probability that the average weight of the 12 eggs in a carton selected at random will be <u>more</u> <u>than</u> 68 grams?

Click Here for Answers

#4

In a study done on the life expectancy of 500 people in a certain geographic region, the mean age at death was 72 years and the standard deviation was 5.3 years.

a. What is the probability that an individual selected at random will be less than 70 years old?

b. If a sample of 50 people from this region are selected, find the probability that the mean life expectancy will be less than 70 years.

[Click Here for Answers]

#1 ANSWERS

1.

   a. This is asking about only the population, so $\frac{300-303}{3} = -1 = .1587 = 15.87\%$

   b. Sample Mean = 303 (because Population Mean = Sample Mean)
   Sample Standard Deviation = $\frac{3}{\sqrt{10}} = .9486$

   c. $$\frac{300 - 303}{\frac{3}{\sqrt{10}}} = -3.16 = .0008 = .08\%$$

#2 ANSWERS

2.

a. Sample Mean = 264 (because Population Mean = Sample Mean)
Sample Standard Deviation =

$$\frac{16}{\sqrt{15}} = 4.13$$

b. $$\frac{250 - 264}{\frac{16}{\sqrt{15}}} = -3.39 = .0003 = .03\%$$

#3 ANSWERS

3.

a. This is asking about only the population, so

$$\frac{68-65}{5} = .6 = .7257 = 72.57\%\ below, so\ 100\% - 72.57 = 27.43\%\ ABOVE$$

b. Sample Mean = 65 (because Population Mean = Sample Mean)
Sample Standard Deviation = $\frac{5}{\sqrt{12}} = 1.44$

c. $$\frac{68-65}{\frac{5}{\sqrt{12}}} = 2.08 = .9812 = 98.12\%\ below, so\ 100\% - 98.12 = 1.88\%\ ABOVE$$

#4 ANSWERS

4.

a. This is asking about only the population, so

$$\frac{70-72}{5.3} = -.38 = .3520 = 35.20\%$$

b. $$\frac{70-72}{\frac{5.3}{\sqrt{50}}} = -.267 = .0038 = .38\%$$